
Can One Get Bivalence from (Tarskian) Truth and Falsity?

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According to one understanding of them, Tarskian principles about truth (and falsity) aim to explicate the core of the classical conception of truth (and falsity), as endorsed by Aristotle and others:

To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true, so that he who says of anything that it is, or that it is not, will say either what is true or what is false. *Metaphysics* (Book IV.7, 1011^a: 26–8)

Timothy Williamson famously offered an argument from these Tarskian principles in favour of *bivalence* — the contention that whatever says something is either true or false — to the effect that denying bivalence in particular cases classically¹ entails a contradiction. This has played a crucial role in the reception of Williamson's case against the main al-

1 I will assume here a classical framework, shared by both supervaluationists and epistemicists. In some cases, however, weaker logics would suffice.

ternative classical view of the nature of vagueness, supervaluationism, and thus in favor of his own epistemic view.

I begin by providing some background on the significance of this issue vis-à-vis the philosophical debate between competing views on the nature of vagueness (section I). I then rehearse Williamson's original argument (section II). Dwelling on (Andjelković & Williamson, 2000), I show that this argument depends on a contentious formulation of the Tarskian principles about truth (and falsity), a formulation which the supervaluationist can reject without jeopardizing the Tarskian insight (section III). In the paper in question, Miroslava Andjelković and Timothy Williamson argue that, even if the appropriate formulation seems to allow for failure of bivalence in borderline cases, this appearance is illusory, once one grants a further (independent) principle involving biconditionals. Finally, I argue that such a formulation is, however, contentious in a similar manner (section IV).

I conclude that the supervaluationist is in a position to block the argument from Tarskian truth (and falsity) in favour of bivalence.

I Bivalence and the Nature of Vagueness

One of the main views on the nature of vagueness — supervaluationism — has it that vagueness is a phenomenon of *semantic indecision*: (roughly) whatever it is that in the thoughts, experiences and practices of language users determines the meaning of expressions, it fails to determine, for vague expressions, any single one from a given range of similarly natural candidates. Each way of ('arbitrarily') fixing what is left semantically indeterminate gives rise to a *precisification* or *sharpening* of the original vague expression. Although all such sharpenings are, by essence, arbitrary to a certain extent, not all of them are *admissible*. In the case of predicates, admissible ones should preserve *clear cases*, both of application and of non-application — Yul Brynner should count for 'is bald,' while Andy García cannot — and they should also preserve *penumbral connections* — 'Whoever is bald is bald,' 'If someone is bald, then so is anyone who is balder,' and so on —.² What one says by means of a vague expression is true, according to this view, if it would

2 Thus sharpenings are, strictly speaking, of the language as a whole, and not of isolated expressions, see (Fine 1975). How to characterize in an explicit satisfactory way the notion of *admissible* constituted by these connections (possibly among others) — though central to a full defense of the view of vagueness as semantic indecision — is not crucial to our present concerns. Notice that 'is admissible' is, of course, itself vague: this is arguably part of what accounts, in this framework, for

be true however one (admissibly) precisifies it — or, as I will put it, if it counts as true according to all admissible sharpenings. And it is false if it counts as false according to all of admissible sharpenings. Otherwise, if there are admissible ways of precisifying it which give rise to truths, but also admissible ways of precisifying it which give rise to falsehood, the vague sentence is indeterminate: neither true nor false.

That is indeed the situation with respect to borderline cases, as the view has it. Take Harry, a borderline case with respect to ‘is bald,’ having exactly 3,833 hairs on his scalp. Whatever it is that in the thoughts, experiences, and practices of language users determines the meaning of expressions, it fails to determine whether someone with this very number of hairs does or does not fall under ‘is bald.’ Thus ‘is bald’ can be admissibly precisified by (let us assume) ‘has at most 3,832 hairs on his scalp,’ but also by ‘has at most 3,834 hairs on his scalp.’³ Hence, ‘Harry is bald’ fails to be true, given that ‘Harry has at most 3,832 hairs on his scalp’ is false, but it also fails to be false, given that ‘Harry has at most 3,834 hairs on his scalp’ is true. Thus the characteristic denial of the principle of bivalence: not everything that says something is either true or false, as borderline cases are indeterminate.

In *Vagueness*, Williamson (1994) offered an argument to the effect that denying bivalence in particular cases classically entails a contradiction. That argument is advanced there as the main positive consideration in favor of his own epistemic view on the nature of vagueness, and it has played a crucial role in the reception of Williamson’s case against supervaluationism, given the denial of bivalence that the view endorses.⁴

the phenomenon of ‘higher-order’ vagueness. Complications arising from this will be set aside here.

- 3 Notice that I am calling ‘sharpenings’ and ‘precisifications’ the (relatively more) precise linguistic expressions themselves, as opposed to the entities these would (determinately) signify. This usage is not uncommon but also not universal. In any case, nothing of substance hinges on this here, and it is congenial with the quantification into sentence position below, see footnote 5.
- 4 (Richard, 2000) argues that, regarding a specific instance of bivalence, one could *reject* it, in the sense of committing oneself to its non-truth, without *denying* it, in the sense of committing oneself to its falsity. (Richard, however, uses ‘denying’ for what, following (Field, 2003), I am calling ‘rejecting.’) This is explicitly considered in (Williamson, 1994). In any case, supervaluationists deny, and not merely reject, instances of bivalence with respect to borderline cases.

II The Argument

The argument assumes the Tarskian principles about truth (and falsity). In *Vagueness*, Williamson (1994, 188) formulated them thus:⁵

$$(T) \forall s \forall c \forall P [\text{Say}(s, c, P) \supset [\text{True}(s, c) \equiv P]]$$

$$(F) \forall s \forall c \forall P [\text{Say}(s, c, P) \supset [\text{False}(s, c) \equiv \neg P]]$$

Under these assumptions, he claims, the supposition of a counterexample to bivalence leads to a contradiction. Bivalence is formulated thus:⁶

$$(B) \forall s \forall c \forall P [\text{Say}(s, c, P) \supset [\text{True}(s, c) \vee \text{False}(s, c)]]$$

Consider the following (for some sentence in place of 'P' — for example, 'Harry is bald'):

$$(0) \text{Say}(s, c, P)$$

$$(1) \neg[\text{True}(s, c) \vee \text{False}(s, c)]$$

$$(2a) \text{True}(s, c) \equiv P$$

$$(2b) \text{False}(s, c) \equiv \neg P$$

5 I have altered the original formulations in terms of utterances so as to match their more recent presentation, which is in terms of sentences in contexts (Andjelković & Williamson, 2000, 215): 'We use the expression $\text{Say}(s, c, P)$ to mean something like: the sentence s as uttered in the context c says (or is used to say) that P (where ' P ' may be replaced by a declarative sentence); in terms of propositions, s in c expresses the proposition that P . We use $\text{True}(s, c)$ (respectively, $\text{False}(s, c)$) to mean that s is true (respectively, false) in c .' Notice that the principles are formulated with the help of quantification into sentence position, see (Andjelković & Williamson, 2000, 216-17) for discussion.

6 This is (classically equivalent to) Andjelković and Williamson's *weak bivalence*:

$$(WB) \forall s \forall c \forall P [\text{Say}(s, c, P) \supset [\neg \text{True}(s, c) \supset \text{False}(s, c)]]$$

As they observed (2000, 218-19), only the right-to-left directions of the embedded biconditionals in (T) and (F) are required for (B). The other two directions suffice in turn for a principle that truth and falsity are mutually exclusive:

$$(ME) \forall s \forall c \forall P [\text{Say}(s, c, P) \supset [\text{False}(s, c) \supset \neg \text{True}(s, c)]]$$

(WB) and (ME) combine into a single principle

$$(SB) \forall s \forall c \forall P [\text{Say}(s, c, P) \supset [\neg \text{True}(s, c) \equiv \text{False}(s, c)]]$$

which they label *strong bivalence*.

$$(3) \neg[\mathbf{P} \vee \neg\mathbf{P}]$$

$$(4) \neg\mathbf{P} \wedge \neg\neg\mathbf{P}$$

A counterexample to (B) provides (0) and (1), whereas (2a) and (2b) come by detaching the consequents of instances of (T) and (F), thanks to (0). (1), (2a) and (2b) yield (3), provided that the biconditionals equate status. And (3) in turn suffices, by one of the De Morgan's laws, for (4), which is a contradiction. (See (Williamson, 1994, 188-90).)

III Truth, Falsity, and Bivalence

The argument as stated depends crucially on the formulations (T) and (F). Andjelković and Williamson have argued that these formulations might be claimed to be inappropriately contentious.

(T) and (F) are too strong to be mere definitions of **True** and **False**, because they are *creative*. That is, (T) has a consequence not involving **True** and (F) has a consequence not involving **False**. More specifically, each of them entails a principle of *uniformity* to the effect (at least in the classical context) that everything said by a given sentence in a given context has the same truth-value. (Andjelković & Williamson, 2000, 225)

This is the entailed principle of uniformity:⁷

$$(U) \forall s \forall c \forall P \forall Q [[\text{Say}(s,c,P) \wedge \text{Say}(s,c,Q)] \supset [P \equiv Q]]$$

But this, as they observe, makes an appeal to (T) and (F) when arguing in favour of (B) dialectically inappropriate:⁸

the anti-epistemicist might reject (U), and therefore both (T) and (F), on the grounds that (U) fails in borderline cases for a vague sentence *s*. Perhaps they will suggest that a vague sentence says many things, corresponding to its different possible sharpenings; in a borderline case, some of these things differ from others in truth-value, contrary to (U). (Andjelković & Williamson, 2000, 226)

7 In fact, (U) exhausts the non-conservative aspect of (T) and (F) in that it is the strongest consequence of the conjunction of (T) and (F) to contain neither **True** nor **False**: if **A** contains neither **True** nor **False** and (T) and (F) together entail **A** then (U) entails **A**, see (Andjelković & Williamson, 2000, 227-28).

8 NB: The problem to come is *not* merely that, if one assumes there are counterexamples to (B), then there would be counterexamples to (T) and (F), as this is, of course, to be expected. In my view, (Pelletier & Stainton, 2003) offer a criticism of this sort, see the discussion of this point in (Williamson, 1994, 190).

The expression 'say,' even in the context of 'what sentences in context say' may say a number of different things. One of them arguably concerns the relation that holds between sentences as used in contexts and certain (precisely specifiable) entities, whose obtaining (or not) is relevant for the truth (and falsity) of sentences in contexts — *state of affairs* (say). There is obviously a great deal to be said about the details. But, for our present concerns, what is crucial to note is that, as Andjelković and Williamson acknowledge, the supervaluationist can sensibly conceive of this relation as that which corresponds to the the relation that holds between vague sentences and their sharpenings.

In other words, and for further reference, Andjelković and Williamson acknowledge that **Say** as figuring in the formulations of the Tarskian principles should at least be compatible with the following supervaluationist-oriented contention:

$$(S) \forall s \forall c \forall P [\text{Say}(s, c, P) \equiv \text{P} \text{ precisifies } s \text{ in } c]$$

Of course, the sense of 'say' of (S), as alluded to above, is not the only sense that 'say' has in English, and plausibly not even the most 'common' sense (if such there is). The supervaluationist has every reason to acknowledge this. For in the sense of 'say' of (S), the different things a vague sentence such as 'Harry is bald' says are arguably true or false (indeed, some true and some false), whereas there is another sense of 'say' in which, the supervaluationist would hold, the sentences says one thing — and that thing fails to be true or false.⁹ The present point is simply that the supervaluationist can sensibly contend that it is the sense of (S) that should be involved in the formulations of the Tarskian principles, and that Andjelković and Williamson are acknowledging as much here.

Andjelković and Williamson then offer the following alternative formulation of the Tarskian insight:¹⁰

$$(T^*) \forall s \forall c [\text{True}(s, c) \equiv [\exists P \text{ Say}(s, c, P) \wedge \forall P [\text{Say}(s, c, P) \supset P]]]$$

9 There might also even be a 'sophisticated' sense of 'say' which satisfies the appropriate formulations of the Tarskian principles *and* the (independent) uniformity principle (U). (Ebbs, 2001) could be seen as arguing that one cannot simply *assume* that borderline sentences *say* in this 'sophisticated' sense. As we are about to see, (Andjelković & Williamson, 2000) do not merely assume this, but explicitly argue in favor of it. However, I will contend, their argument is not ultimately satisfactory.

10 The following are labelled respectively '(TDEF2*)' and '(FDEF2*)' in (Andjelković & Williamson, 2000, 230).

$$(F^*) \forall s \forall c [\text{False}(s,c) \equiv [\exists P \text{ Say}(s,c,P) \wedge \forall P [\text{Say}(s,c,P) \supset \neg P]]]$$

As they contend, these formulations seem congenial to supervaluationism, and do not jointly entail (U), nor thereby bivalence. Andjelković and Williamson conclude that the supervaluationist seems to be in a position to block the argument from principles about (Tarskian) truth and falsity in favor of bivalence:

If borderline cases falsify (U), they are classified [by (T*) and (F*)] as involving truth-value gaps. (Andjelković & Williamson, 2000, 230)

IV Against Uniformity

The formulations of the Tarskian principles about truth and falsity seem to allow for failure of bivalence in borderline cases, as such cases may appear to falsify the principle of uniformity that the argument for bivalence requires. One of the aims of (Andjelković & Williamson, 2000) is, nonetheless, to argue that this appearance is illusory.

What they show is that (T*) in conjunction with independent principles entails (U). These concern material biconditionals connecting sentences *s* and *t* into a sentence *Est* (Andjelković & Williamson, 2000, 233):¹¹

$$(E1) \forall s \forall t \forall c \forall P \forall Q [[\text{Say}(s,c,P) \wedge \text{Say}(t,c,Q)] \supset \text{Say}(\text{Est},c,P \equiv Q)]$$

$$(E2) \forall s \forall c \forall P [\text{Say}(s,c,P) \supset \text{True}(\text{Ess},c)]$$

They contend that

[s]upervaluationists would certainly accept (E2), since every sharpening of *Ess* is true; it is a classical tautology. (Andjelković & Williamson, 2000, 234)

11 The argument is the following. Consider:

$$(1) \forall s \forall c \forall P \forall Q [[\text{Say}(s,c,P) \wedge \text{Say}(s,c,Q)] \supset \text{Say}(\text{Ess},c,P \equiv Q)]$$

$$(2) \forall s \forall c \forall P \forall Q [\text{Say}(\text{Ess},c,P \equiv Q) \supset [\text{True}(\text{Ess},c) \supset [P \equiv Q]]]$$

$$(3) \forall s \forall c \forall P \forall Q [[\text{Say}(s,c,P) \wedge \text{Say}(s,c,Q)] \supset [\text{True}(\text{Ess},c) \supset [P \equiv Q]]]$$

(1) is a special case of (E1) and (2) is entailed by (T*). (1) and (2) entail (3), which in turn, in the presence of (E2), suffice for (U), see (Andjelković & Williamson, 2000, 233-4).

According to them, however, the fact that (T*), (E1), and (E2) entail (U) is what shows the illusory character of the supervaluationist-friendly seeming (T*) and (F*):

Although (T*) and (F*) appear to invite a supervaluationist treatment of vagueness, they do not really do so. Such a treatment would involve the denial of (U). The foregoing argument shows that that in turn would require the supervaluationist to deny (E1). (Andjelković & Williamson, 2000, 234)

Now, it seems to me that rather than being ‘required’ by the argument to deny (E1), the supervaluationist has good motivation for so doing — and indeed of the very same sort as the one favoring (E2), which they have just mentioned.

In the previous discussion, Andjelković and Williamson have granted that (T*) appears to invite a supervaluationist treatment of vagueness, by allowing that a sentence may be said to say — in the relevant sense to figure in the formulation of the Tarskian principles — their different sharpenings. This is what I have proposed to state by (S). It is certainly the case that, under this understanding, the supervaluationist will comply with (E2), given that everything that Ess says, under this understanding, holds. Every sharpening of Ess is true, as they observe, and this is so even if we take *s* to be a vague sentence, such as ‘Harry is bald.’ That instances of classical tautologies such as ‘Harry is bald if and only if Harry is bald’ are reckoned to be penumbral truths is precisely one of the main motivations for supervaluationism, (Fine, 1975). But this in turn requires, as we saw, that admissible precisifications be appropriately constrained. There are sharpenings counting *s* as true and sharpenings counting *s* as false — but no sharpening counting Ess as false. Letting **P** and **Q** be of the former and the latter sort, respectively, we have a counterexample to (E1): **Say**(*s*,*c*,**P**) and **Say**(*s*,*c*,**Q**) hold, yet **Say**(**Ess**,*c*,**P** \equiv **Q**) does not. To illustrate, take borderline bald Harry again, having exactly 3,833 hairs on his scalp. As we saw, both the true ‘Harry has at most 3,834 hairs on his scalp’ and the false ‘Harry has at most 3,832 hairs on his scalp’ *do* admissibly precisify ‘Harry is bald.’ However, ‘Harry is bald iff Harry is bald’ is a penumbral truth, which all admissible sharpenings are required to respect. Hence, the false ‘Harry has at most 3,834 hairs on his scalp iff Harry has at most 3,832 hairs on his scalp’ is *not* an admissible precisification of ‘Harry is bald iff Harry is bald.’ Therefore, given the concession about (S) Andjelković and Williamson made in discussing (U), (E1) fails.

My point is, therefore, that Andjelković and Williamson’s result — that assuming (T*) and (E2) the truth of (E1) would require the truth of (U) — can be seen as a way of articulating precisely the supervalu-

ationist rationale against (E1).¹² According to them, a supervaluationist rejection of (E1) is

too high a price to pay, for it destroys our conception of what biconditionals say. (Andjelković & Williamson, 2000, 234)

As we have seen, however, it is (E1) that is incompatible with our conception of what biconditionals say — in the relevant sense of ‘say’ articulated in (S).

Conclusion

My response to Andjelković and Williamson’s argument exploits the concession they made concerning (S), allowing the supervaluationist to use the notion of admissible sharpening to explicate the sense of **Say** as it figures in the formulations of the Tarskian principles about truth and falsity. Andjelković and Williamson remark that

[s]uch an explanation is only as clear as the notion of an admissible sharpening. Supervaluationists have great difficulty in giving an adequate account of that notion in such a way that does not reduce to something epistemic. (Andjelković & Williamson, 2000, 234)

This is a huge difficulty, of course.¹³ But it is a different one from that having to do with the alleged incompatibility between rejecting bivalence and respecting Tarskian principles about truth (and falsity).¹⁴

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12 If I understand it right, the response offered in (García-Carpintero, 2007) may turn out to be ultimately of the same kind.

13 See also (Williamson, 1999) for a similar situation concerning higher-order vagueness.

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